

Sandip Foundation's
Sandip Institute of Technology & Research Centre, Nashik
S. E. III : Mathematics
UNIT 2 : PART I : Assignment II : Fourier Transform

Q.1. Find the Fourier transform of the following functions :

$$(i) \ f(x) = \sin x, \ 0 < x < \pi, \quad (ii) \ f(x) = \sin x, \ 0 < x < \pi \\ = 0, \text{ otherwise.} \quad = 0, \text{ otherwise.}$$

Q.2. Find the sine and cosine transforms of :

$$f(x) = x, \quad 0 \leq x \leq 1, \\ = 2 - x, \quad 1 \leq x \leq 2 \\ = 0, \quad x \notin [0, 2]$$

Q.3. What is the function $f(x)$ whose Fourier cosine transform is $\frac{\sin ax}{x}$?

Q.4. Find the inverse sine transform of $\frac{e^{-ax}}{x}$.

Q.5. Find the Fourier transform of $f(x) = 1 - x^2, \ |x| \leq 1, \\ = 0, \ |x| > 1.$

Hence evaluate $\int_0^\infty \left(\frac{x \cos x - \sin x}{x^3} \right) \cos \frac{x}{2} dx.$

Q.6. Find the Fourier integral representation of $f(x) = 1, \ |x| \leq a \\ = 0, \ |x| > a$

and hence show that $\int_0^\infty \frac{\sin ax}{x} dx = \frac{\pi}{2}.$

Q.7. Using Fourier integral representation, show that

$$\int_0^\infty \frac{\lambda^3 \sin \lambda x}{\lambda^4 + 4} dx = \frac{\pi}{2} e^{-x} \cos x \quad (x > 0)$$

Q.8. Solve the following integral equations :

$$(i) \int_0^\infty f(x) \sin \lambda x dx = 1, \quad 0 \leq x < 1 \\ = 2, \quad 1 \leq x < 2 \\ = 0, \quad x \geq 2.$$

$$(ii) \int_0^\infty f(x) \cos \lambda x dx = 1 - \lambda, \quad 0 \leq \lambda < 1 \\ = 0, \quad \lambda \geq 1$$